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SAMPLE PAPER TEST 06 (2018-19) (SAMPLE ANSWERS)

SUBJECT: MATHEMATICS **MAX. MARKS : 80**

CLASS : X **DURATION : 3 HRS**

General Instruction:

(i) **All** questions are compulsory.

(ii) This question paper contains **30** questions divided into four Sections A, B, C and D.

(iii) **Section A** comprises of 6 questions of **1 mark** each. **Section B** comprises of 6 questions of **2 marks** each. **Section C** comprises of 10 questions of **3 marks** each and **Section D** comprises of 8 questions of **4 marks** each.

(iv) There is no overall choice. However, an internal choice has been provided in two questions in 1 mark each, two questions in 2 marks each, four questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of Calculators is not permitted

SECTION – A

Questions 1 to 6 carry 1 mark each.

1. Write whether $\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}}$ on simplification gives an irrational or a rational number.

Ans:

For writing $\frac{6\sqrt{5}+6\sqrt{5}}{2\sqrt{5}}$ $= 6$ which is rational

2. If x = a, y = b is the solution of the pair of equations x – y = 2 and x + y = 4, find the values of a and b.

Ans:

Solving for x and y and getting $x = 3, y = 1$

$\therefore \quad a = 3, b = 1$

3. If one root of $5x^2 + 13x + k = 0$ is the reciprocal of the other root, then find value of k.

Ans:

Let α and $\frac{1}{\alpha}$ be the root

$\therefore \quad \alpha \cdot \frac{1}{\alpha} = \frac{k}{5} = 1 \Rightarrow \quad k = 5$

OR

If arithmetic mean of two numbers a and b is 8 and ab = 9, find a quadratic equation whose roots are a and b.

Ans: a + b = 16, ab = 9

Quadratic equation is $x^2 – 16x + 9 = 0$

4. If $\Delta ABC \sim \Delta QRP$, $\frac{ar(\Delta ABC)}{ar(\Delta QRP)} = \frac{9}{4}$, and BC = 15 cm, then find PR.

Ans:

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta QRP)} = \left(\frac{BC}{RP}\right)^2 \Rightarrow \quad \frac{9}{4} = \left(\frac{15}{PR}\right)^2 \Rightarrow \quad PR = 10\,\text{cm}$$

5. Two cubes have their volumes in the ratio 1 : 27. Find the ratio of their surface areas.
Ans:

$$\frac{a^3}{A^3} = \frac{1}{27} \Rightarrow \frac{a}{A} = \frac{1}{3}$$

$$\text{Ratio of sufrace area} = \frac{6a^2}{6A^2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

OR

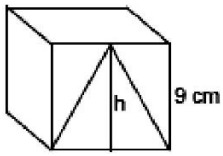
Find the volume of the largest right circular cone that can be cut out of a cube whose edge is 9 cm? [Use = 22/7]
Ans:

Radius of cone $= \frac{1}{2} \times$ edge of cube $= \frac{9}{2}$ cm

Height of cone, $h = 9$ cm

Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \frac{9}{2} \times \frac{9}{2} \times 9$

$= 190.928 \text{ cm}^3 = 190.93 \text{ cm}^3$

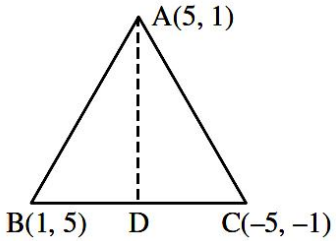


6. A(5, 1); B(1, 5) and C(–3, –1) are the vertices of ΔABC. Find the length of median AD.
Ans:

Coordinates of D are (–1, 2)

$$AD = \sqrt{(5+1)^2 + (1+2)^2}$$

$$= \sqrt{37} \text{ units}$$



SECTION – B

Questions 6 to 12 carry 2 marks each.

7. Given that $\sqrt{3}$ is an irrational number, prove that $(2 + \sqrt{3})$ is an irrational number.
Ans:

Let $2+\sqrt{3}$ be a rational number.

$$\Rightarrow \quad 2+\sqrt{3} = \frac{p}{q}, \quad p, q \in I, q \neq 0$$

$$\Rightarrow \quad \sqrt{3} = \frac{p}{q} - 2 = \frac{p-2q}{q}$$

$\frac{p-2q}{q}$ is rational $\Rightarrow \sqrt{3}$ is rational number which is a contraduction

$2+\sqrt{3}$ is irrational number

OR

Find pairs of natural numbers whose least common multiple is 78 and the greatest divisor is 13.
Ans:

Let numbers are $13x$ and $13y$ where x and y are coprime.

$\therefore$ LCM $= 13 \times x \times y = 78$

$x \times y = 6$

$\therefore$ Possible values of x and y

$x = 1, y = 6, x = 2, y = 3$

$\therefore$ Pairs of natural numbers are 78 and 13 or 26 and 39.

8. X is a point on the side BC of ΔABC. XM and XN are drawn parallel to AB and AC respectively meeting AB in N and AC in M. MN produced meets CB produced at T. Prove that $TX^2 = TB \times TC$

Ans:

$\Delta TXN \sim \Delta TCM$

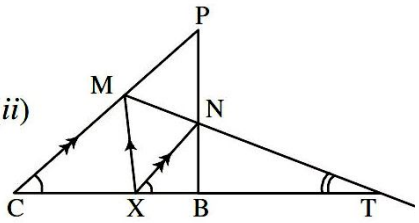
$$\Rightarrow \quad \frac{TX}{TC} = \frac{XN}{CM} = \frac{TN}{TM} \quad \Rightarrow TX \times TM = TC \times TN \ ....(i)$$

Again, $\Delta TBN \sim \Delta TXM$

$$\Rightarrow \quad \frac{TB}{TX} = \frac{BN}{XM} = \frac{TN}{TM} \quad \Rightarrow \quad TM = \frac{TN \times TX}{TB} \ ...(ii)$$

using (ii) in (i), we get

$$TX^2 \times \frac{TN}{TB} = TC \times TN \quad \Rightarrow \quad TX^2 = TC \times TB$$



9. Find the linear relation between x and y such that P(x, y) is equidistant from the points A(1, 4) and B(–1, 2).

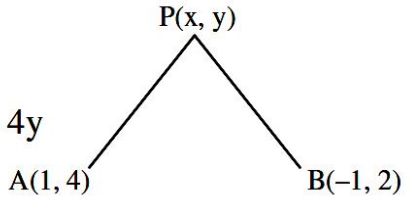
Ans:

$$PA = PB \Rightarrow PA^2 = PB^2$$

$$\Rightarrow \quad (x-1)^2 + (y-4)^2 = (x+1)^2 + (y-2)^2$$

$$\Rightarrow \quad x^2 + 1 - 2x + y + 16 - 8y = x^2 + 1 + 2x + y^2 + 4 - 4y$$

$$\Rightarrow \quad x + y - 3 = 0$$



OR

Find the ratio in which line formed by joining (–1, 1) and (5, 7) is divided by the line x + y = 4.

Ans:

Let line $x + y = 4$ divides the line joining the points (– 1, 1) and (5, 7) at C in the ratio k : 1

$\therefore$ Coordinates of C are $\left(\frac{5k-1}{k+1}, \frac{7k+1}{k+1}\right)$,

$\because$ C lies on the line $x + y = 4$

$$\Rightarrow \frac{5k-1}{k+1} + \frac{7k+1}{k+1} = 4 \quad \Rightarrow \quad k = \frac{1}{2}.$$

Hence ratio 1 : 2.

10. In the below figure, ABC is a triangle in which $\angle B = 90^\circ$, BC = 48 cm and AB = 14 cm. A circle is inscribed in the triangle, whose centre is O. Find radius r of in-circle.

Ans:

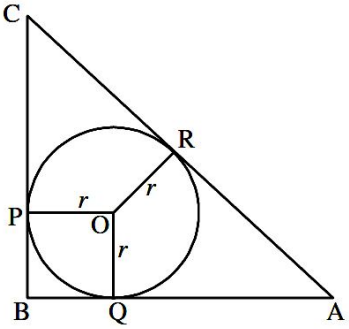
$$AC = \sqrt{AB^2 + BC^2}$$

$$= \sqrt{14^2 + 48^2} = \sqrt{2500} = 50\,\text{cm}$$

$\angle OQB = 90^\circ \Rightarrow OPBQ$ is a square

$$\Rightarrow \quad BQ = r, QA = 14 - r = AR$$

Again $PB = r$, $PC = 48 - r \Rightarrow RC = 48 - r$



11. A, B, C are interior angles of ΔABC. Prove that $\cos ec\left(\frac{A+B}{2}\right)=\sec\frac{C}{2}$

Ans:

$A + B + C = 180°$

$$\Rightarrow \quad \frac{A+B}{2} = 90° - \frac{C}{2} \quad \Rightarrow \quad \text{cosec}\left(\frac{A+B}{2}\right) = \text{cosec}\left(90° - \frac{C}{2}\right) = \sec\frac{C}{2}$$

12. A right circular cylinder and a cone have equal bases and equal heights. If their curved surface areas are in the ratio 8 : 5, show that the ratio between radius of their bases to their height is 3 : 4.

Ans:

Let r be the radii of bases of cylinder and cone and h be the height

Slant height of cone $= \sqrt{r^2 + h^2}$

$$\therefore \quad \frac{2\pi rh}{\pi r\sqrt{r^2+h^2}} = \frac{8}{5}$$

$$\frac{h}{\sqrt{r^2+h^2}} = \frac{4}{5} \quad \Rightarrow \quad \frac{h^2}{r^2+h^2} = \frac{16}{25} \quad \Rightarrow \quad 25h^2 = 16r^2 + 16h^2$$

$$\Rightarrow \quad 9h^2 = 16r^2$$

$$\Rightarrow \quad \frac{r^2}{h^2} = \frac{9}{16} \Rightarrow \frac{r}{h} = \frac{3}{4}$$

SECTION – C

Questions 13 to 22 carry 3 marks each.

13. Using Euclid's division algorithm find the HCF of the numbers 867 and 255.

Ans:

$867 = 255 \times 3 + 102$

$255 = 102 \times 2 + 51$

$102 = 51 \times 2 + 0$

$\Rightarrow \quad HCF = 51$

14. Divide 27 into two parts such that the sum of their reciprocals is $\frac{3}{20}$

Ans:

Let two parts be x and 27 – x

$$\therefore \quad \frac{1}{x} + \frac{1}{27-x} = \frac{3}{20} \quad \Rightarrow \quad x^2 - 27x + 150 = 0$$

$$\Rightarrow \quad (x - 15)(x - 12) = 0 \quad \Rightarrow \quad x = 12 \text{ or } 15$$

$\therefore$ The two parts are 12 and 15

15. In an A.P if sum of its first n terms is $3n^2 + 5n$ and its kth term is 164, find the value of k.

Ans:

$S_1 = 3.1^2 + 5.1 = 8 = a_1$

$S_2 = 3.2^2 + 5.2 = 22 = a_1 + a_2$

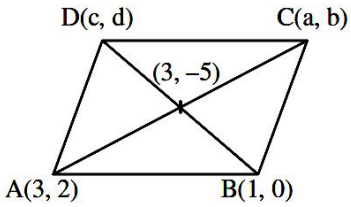
$a_2 = 22 - 8 = 14 \Rightarrow d = 6$

$t_k = 164 \Rightarrow 8 + (k - 1)6 = 164 \quad \Rightarrow \quad k = 27$

16. If coordinates of two adjacent vertices of a parallelogram are (3, 2), (1, 0) and diagonals bisect each other at (2, –5), find coordinates of the other two vertices.

Ans:

Let the coordinates of C and D be (a, b) and (c, d)



$\therefore \quad \frac{3+a}{2} = 2 \Rightarrow a = 1 \quad \text{and} \quad \frac{2+b}{2} = -5 \Rightarrow b = -12$

$\text{Also} \quad \frac{c+1}{2} = -5 \Rightarrow c = 3$

$\text{and} \quad \frac{d+0}{2} = -5 \Rightarrow d = -10$

Coordinate of C and D are (1, –12) and (3, –10)

OR

If the area of triangle with vertices (x, 3), (4, 4) and (3, 5) is 4 square units, find x.

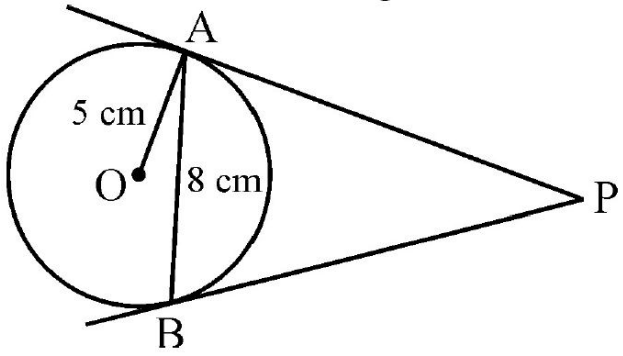
Ans:

$\text{Ar}(\Delta ABC) = 4$

$\Rightarrow \quad \frac{1}{2}[x(4-5) + 4(5-3) + 3(3-4)] = 4$

$\Rightarrow \quad (-x + 5) = 8 \quad \Rightarrow \quad -x + 5 = 8 \quad \Rightarrow \quad x = -3$

17. In below figure, AB is a chord of length 8 cm of a circle of radius 5 cm. The tangents to the circle at A and B intersect at P. Find the length of AP.



(NCERT Circles Example-3, page – 212)

OR

Prove that the lengths of tangents drawn from an external point to a circle are equal.

18. Construct a triangle with sides 6 cm, 8 cm and 10 cm. Construct another triangle whose sides are 3/5 the corresponding sides of original triangle.

19. Prove that $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

(NCERT Introduction to Trigonometry Exercise – 8.4, Q5 (x))

OR

Evaluate: $\frac{\cos 58^0}{\sin 32^0} + \frac{\sin 22^0}{\cos 68^0} - \frac{\cos 38^0 \cos ec 52^0}{\sqrt{3}\left(\tan 18^0 \tan 35^0 \tan 60^0 \tan 72^0 \tan 55^0\right)}$

Ans:

$\frac{\cos 58°}{\sin 32°} + \frac{\sin 22°}{\cos 68°} - \frac{\cos 38° \operatorname{cosec} 52°}{\sqrt{3}(\tan 18° \tan 35° \tan 60° \tan 72° \tan 55°)}$

$$= \left(\frac{\cos 58°}{\sin(90-58°)} + \frac{\sin 22°}{\cos(90-22°)}\right) - \frac{\cos 38° \operatorname{cosec}(90-38)°}{\sqrt{3}(\tan 18° \tan 35° \cdot \sqrt{3} \cdot \cot 18° \cot 35°)}$$

$$= 1+1-\frac{\cos 38° \sec 38°}{3.1} \quad = 2-\frac{1}{3} = \frac{5}{3}$$

20. The short and long hands of a clock are 4 cm and 6 cm long respectively. Find the sum of distances travelled by their tips in 48 hours.

Ans:

Distance travelled by short hand in 48 hours $= 4 \times 2\pi \times 4 \text{ cm} = 32\pi \text{ cm}$

Distance travelled by long hand in 48 hours $= 48 \times 2\pi \times 6\text{cm} = 576\pi \text{ cm}$

Total distance travelled $= (32\pi + 576\pi)\text{cm} \quad = 608\pi \text{ cm}$

OR

The side of a square is 10 cm. Find the area between inscribed and circumscribed circles of the square.

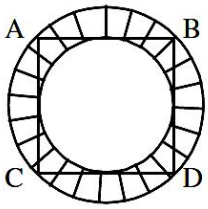
Ans:

Radius of inner circle $= 5$ cm

Radius of outer circle $= 5\sqrt{2}$ cm

Required area = Area of outer circle – Area of inner circle

$\Rightarrow \quad [(5\sqrt{2})^2 - 5^2] = 25\pi \text{ cm}^2$



21. If $\sin(A + 2B) = \frac{\sqrt{3}}{2}$ and $\cos(A + 4B) = 0$, $A > B$, and $A + 4B \leq 90°$, then find A and B.

Ans:

$\sin(A+2B) = \frac{\sqrt{3}}{2} \Rightarrow A + 2B = 60°$

$\cos(A+4B) = \Rightarrow A + 4B = 90°$

Solving, we get $A = 30°, B = 15°$

22. By changing the following frequency distribution 'to less than type' distribution, draw its ogive.

Classes	0 – 15	15 – 30	30 – 45	45 – 60	60 – 75
Frequency	6	8	10	6	4

Ans:

Classes	**Frequency**	**Classes**	**Cumulative frequency**
0-15	6	Less than 15	6
15-30	8	Less than 30	14
30-45	10	Less than 45	24
45-60	6	Less than 60	30
60-75	4	Less than 75	34

SECTION – D

Questions 23 to 30 carry 4 marks each.

23. For what values of m and n the following system of linear equations has infinitely many solutions. $3x + 4y = 12$ and $(m + n)x + 2(m – n)y = 5m – 1$

Ans:

For infinitely many solutions.

$$\frac{3}{m+n} = \frac{4}{2(m-n)} = \frac{-12}{-(5m-1)}$$

$$\frac{3}{m+n} = \frac{4}{2(m-n)} \Rightarrow m - 5n = 0 \qquad ...(1)$$

$$\frac{4}{2(m-n)} = \frac{12}{5m-1} \Rightarrow m - 6n = -1 \qquad ...(2)$$

Solving (1) and (2) we get, $m = 5, n = 1$

24. Obtain all zeroes of $3x^4 - 15x^3 + 13x^2 + 25x - 30$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

Ans:

$p(x) = 3x^4 - 15x^3 + 13x + 25x - 30$

$x - \sqrt{\frac{5}{3}}$ and $x + \sqrt{\frac{5}{3}}$ are factors of $p(x)$

$\Rightarrow \quad x^2 - \frac{5}{3}$ or $\frac{(3x^2 - 5)}{3}$ is a factor of $p(x)$

$$p(x) = \frac{(3x^2 - 5)}{3}(x^2 - 5x + 6) = \frac{1}{3}(3x^2 - 5)(x - 3)(x - 2)$$

$\therefore$ Zeroes of $p(x)$ are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, 2$ and 3

25. A faster train takes one hour less than a slower train for a journey of 200 km. If the speed of slower train is 10 km/hr less than that of faster train, find the speeds of two trains.

Ans:

Let the speed of faster train be x km/hr

$\therefore$ Speed of slower train = $(x - 10)$ km/hr

$$\frac{200}{x-10} - \frac{200}{x} = 1$$

$\Rightarrow \quad x^2 - 10x - 2000 = 0 \quad \Rightarrow \quad (x - 50)(x + 40) = 0$

$x = 50, -40$ rejected

$\therefore$ Speed of faster train = 50 km / hr

Speed of slower train = 40 km / hr

OR

Solve for x: $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}, a \neq 0, b \neq 0, x \neq 0$

Ans:

$$\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x} \Rightarrow \frac{1}{a+b+c} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$$

$$\frac{-(a+b)}{x(a+b+x)} = \frac{a+b}{ab}$$

$\Rightarrow \quad x^2 + (a + b)x + ab = 0$

$(x + a)(x + b) = 0 \Rightarrow x = -a, -b$

26. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

27. The angle of elevation of the top of a hill at the foot of a tower is 60º and the angle of depression from the top of tower to the foot of hill is 30º. If tower is 50 metre high, find the height of the hill.

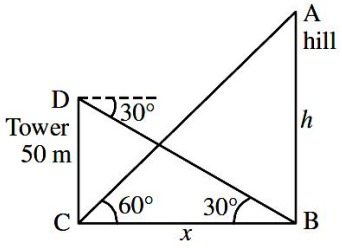
Ans:

$\ln \Delta ABC, \frac{h}{x} = \tan 60° \Rightarrow h = x\sqrt{3}$

$\ln \Delta BCD, \frac{50}{x} = \tan 30° \Rightarrow x = 50\sqrt{3}$

$\therefore \quad h = 150$

$\therefore \quad$ height of hill $= 150$ m



OR

Two poles of equal heights are standing opposite to each other on either side of the road which is 80 m wide. From a point in between them on the road, the angles of elevation of the top of poles are 60º and 30º respectively. Find the height of the poles and the distances of the point from the poles. **(NCERT Exercise 9.1 Q10)**

28. A man donates 10 aluminum buckets to an orphanage. A bucket made of aluminum is of height 20 cm and has its upper and lowest ends of radius 36 cm and 21 cm respectively. Find the cost of preparing 10 buckets if the cost of aluminum sheet is Rs. 42 per 100 cm^2. Write your comments on the act of the man.

Ans:

Surface area of bucket $= \pi(r_1 + r_2)l + \pi r_1^2$

$l = \sqrt{h^2 + (r_2 - r_1)^2} = \sqrt{20^2 + (36-21)^2} = \sqrt{625} = 25$ cm

$\therefore$ Surface area of 1 bucket $= \frac{22}{7}[(36+21)\times 25 + 21^2] = \frac{22}{7}\times 1866\ cm^2$

Surface area of 10 buckets $= \frac{22}{7}\times 18660\ cm^2$

Cost of aluminium sheet $= ₹\ \frac{22}{7}\times\frac{18660\times 42}{100} = ₹\ 24631.20$

29. Find the mean and mode for the following data :

Classes	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Frequency	4	8	10	12	10	4	2

Ans: Mean = 42.2, Mode = 45

30. A box contains cards numbered from 1 to 20. A card is drawn at random from the box. Find the probability that number on the drawn card is

(i) a prime number (ii) a composite number (iii) a number divisible by 3

Ans: $(i)\frac{8}{20} = \frac{2}{5}$ $(ii)\frac{11}{20}$ $(iii)\frac{6}{20} = \frac{3}{10}$

OR

The King, Queen and Jack of clubs are removed from a pack of 52 cards and then the remaining cards are well shuffled. A card is selected from the remaining cards. Find the probability of getting a card

(i) of spade (ii) of black king (iii) of club (iv) of jacks

Ans: $(i)\frac{13}{49}$ $(ii)\frac{1}{49}$ $(iii)\frac{10}{49}$ $(iv)\frac{3}{49}$